Neural Network Embedding of the Over-Dispersed Poisson Reserving Model

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Idea

- CANN (Combined Actuarial Neural Network) approach
- Embedding of cross-classified over-dispersed Poisson (ccODP) reserving model into neural network architecture
- Starting point of neural network calibration: ccODP model

$\Longrightarrow$ Learning model structure beyond ccODP model (boosting)

Example Data

- Simulated from Individual Claims History Simulation Machine
- $1 \leq i \leq I$: accident years, $0 \leq j \leq J$: development delays
- Aggregated (incremental) payments $Y_{i,j}$ for all claims in LoB 1:

Accident year i	Development delay j											
	0	1	2	3	4	5	6	7	8	9	10	11
1	9'416	4'850	1'596	871	594	446	322	242	188	177	159	130
2	9'822	5'293	1'826	1'026	657	457	364	292	228	191	146	
3	9'613	4'903	1'665	970	594	443	325	263	212	176		
4	9'788	5'250	1'823	1'086	744	550	431	303	226			
5	9'955	5'722	2'089	1'159	791	558	458	354				
6	10'453	6'122	2'214	1'311	859	630	497					
7	11'130	6'476	2'401	1'356	890	677						
8	11'268	6'629	2'504	1'493	1'008							
9	11'475	6'953	2'648	1'478								
10	12'172	7'084	2'746									
11	12'816	8'028										
12	13'239											

Cross-Classified Over-Dispersed Poisson (ccODP) Model

- ODP model: $Y_{i,j}/\phi \overset{\text{ind.}}{\sim} \text{Poi}(\mu_{i,j}/\phi), \quad \phi > 0$
- Cross-classification: $\log \mu_{i,j} = \alpha_i + \beta_j$

$$\Longrightarrow \mathbb{E}[Y_{i,j}] = \text{Var}(Y_{i,j})/\phi = \mu_{i,j} = \exp\{\alpha_i + \beta_j\}$$

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- Minimize Poisson deviance statistics $\Longrightarrow$ MLEs $(\widehat{\alpha}_i)_i, (\widehat{\beta}_j)_j$

- Estimates: $\widehat{Y}_{i,j}^{\text{ODP}} = \widehat{\mu}_{i,j}^{\text{ODP}} = \exp\left\{\widehat{\alpha}_i + \widehat{\beta}_j\right\}$

- ODP reserves $= \sum_{i+j>I} \widehat{\mu}_{i,j}^{\text{ODP}} =$ Chain-ladder (CL) reserves

Results

- 6 LoBs from Individual Claims History Simulation Machine
- Results:

		LoB 1	LoB 2	LoB 3	LoB 4	LoB 5	LoB 6	total
(i)	true claims reserves	39'689	37'037	16'878	71'630	72'548	31'117	268'899
(ii)	CL/ccODP reserves	38'569	35'460	15'692	67'574	70'166	29'409	256'870
(iii)								
(iv)								
(v)	bias CL/ccODP	-2.8%	-4.3%	-7.0%	-5.7%	-3.3%	-5.5%	-4.5%
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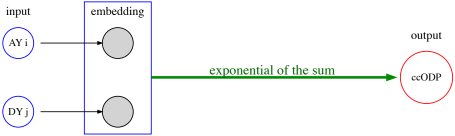
- Question: Can we do better?

$\Longrightarrow$ Embed ccODP model into neural network architecture

ccODP Model as Neural Network



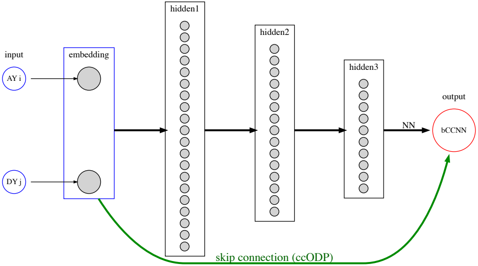
- Input layer: $(i,j) \in \{1,\ldots,I\} \times \{0,\ldots,J\}$
- Embedding layers:

$$\alpha(\cdot):\ \{1,\ldots,I\} \to \mathbb{R}, \qquad i \mapsto \alpha(i) = \widehat{\alpha}_i,$$

$$\beta(\cdot):\ \{0,\ldots,J\} \to \mathbb{R}, \qquad j \mapsto \beta(j) = \widehat{\beta}_j.$$

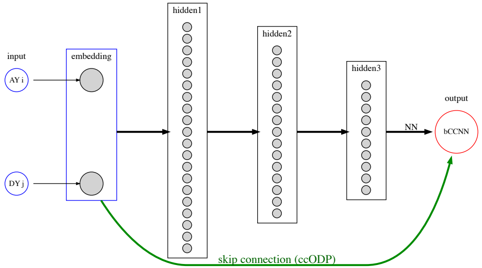
- ccODP: $\widehat{\mu}_{i,j}^{\mathrm{ODP}} = \exp\left\{\widehat{\alpha}_i + \widehat{\beta}_j\right\}$

Neural Network Embedding (1/3)



- Neural network: (non-linear) parametric regression function
- Input layer: $(i, j) \in \{1, \ldots, I\} \times \{0, \ldots, J\}$
- Embedding layers: $(i, j) \mapsto \left(\widehat{\alpha}_i, \widehat{\beta}_j\right)$

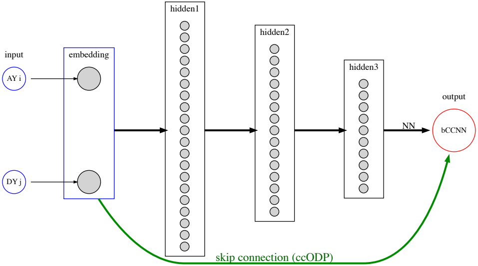
Neural Network Embedding (2/3)



- Three hidden layers with $(q_1, q_2, q_3) = (20, 15, 10)$
- First hidden layer: $\boldsymbol{z}^{(1)} = \left(z_1^{(1)}, \ldots, z_{q_1}^{(1)}\right) \in \mathbb{R}^{q_1}$, where

$$z_l^{(1)} = \tanh\left(b_l^{(1)} + w_{l,1}^{(1)}\,\widehat{\alpha}_i + w_{l,2}^{(1)}\,\widehat{\beta}_j\right) \in (-1, 1)$$

Neural Network Embedding (3/3)

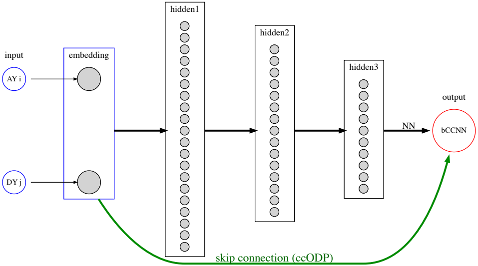


- Second hidden layer: $\boldsymbol{z}^{(2)} = \left(z_1^{(2)}, \ldots, z_{q_2}^{(2)}\right) \in \mathbb{R}^{q_2}$, where

$$z_l^{(2)} = \tanh\left(b_l^{(2)} + \langle \boldsymbol{w}_l^{(2)}, \boldsymbol{z}^{(1)} \rangle\right)$$

- Third hidden layer: $\boldsymbol{z}^{(3)} \in \mathbb{R}^{q_3}$

Blended Cross-Classified Neural Network (bCCNN)



- **Output:** $\mu_{i,j}^{\mathrm{bCCNN}} = \exp\left\{ \qquad\qquad b + \langle \boldsymbol{w}, \boldsymbol{z}^{(3)}(i,j)\rangle \right\}$

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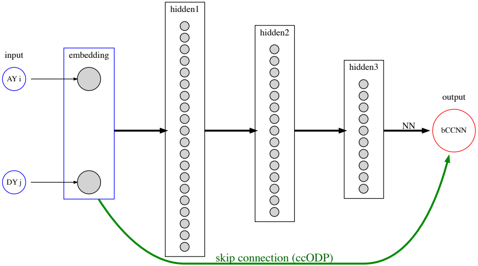

- Output: $\mu_{i,j}^{\mathrm{bCCNN}} = \exp\left\{\widehat{\alpha}_i + \widehat{\beta}_j + b + \langle \boldsymbol{w}, \boldsymbol{z}^{(3)}(i,j)\rangle\right\}$

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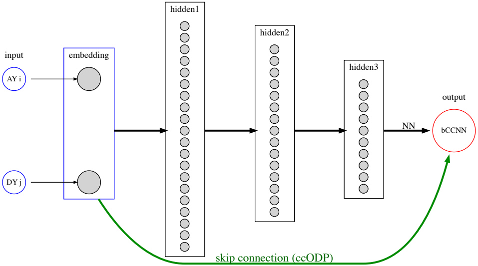

- Output: $\mu_{i,j}^{\text{bCCNN}} = \exp\left\{\widehat{\alpha}_i + \widehat{\beta}_j + b + \langle \boldsymbol{w}, \boldsymbol{z}^{(3)}(i,j)\rangle\right\}$
- Initialization: $b = 0,\ \boldsymbol{w} = \boldsymbol{0} \implies \mu_{i,j}^{\text{bCCNN}} = \exp\left\{\widehat{\alpha}_i + \widehat{\beta}_j\right\} = \widehat{\mu}_{i,j}^{\text{ODP}}$

$\implies$ Starting point of neural network calibration: ccODP model

Neural Network Calibration

- Neural network parameter: $\boldsymbol{\theta} \in \mathbb{R}^q$ with $q = 547$
- Minimize Poisson deviance statistics $\mathcal{L}(\boldsymbol{\theta})$ with gradient descent:

$$\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} - \rho \nabla_{\boldsymbol{\theta}} \mathcal{L}(\boldsymbol{\theta}), \qquad \rho > 0$$

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- Idea: Split claims $\Longrightarrow$ training triangle and validation triangle

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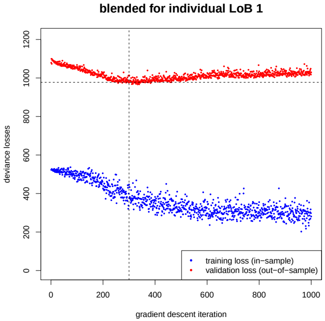

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(v)	bias CL/ccODP	-2.8%	-4.3%	-7.0%	-5.7%	-3.3%	-5.5%	-4.5%
(vi)	bias bCCNN	-1.1%	-3.1%	-6.3%	-2.0%	-2.2%	-1.4%	-2.3%
(vii)								

Multiple LoB Model

- Input layer: $(i, j, m) \in \{1, \dots, I\} \times \{0, \dots, J\} \times \{1, \dots, 6\}$

- Embedding layers:

$$\alpha(\cdot) : \{1, \dots, I\} \to \mathbb{R}^6, \qquad i \mapsto \alpha(i) = \left(\widehat{\alpha}_{i|1}, \dots, \widehat{\alpha}_{i|6}\right),$$

$$\beta(\cdot) : \{0, \dots, J\} \to \mathbb{R}^6, \qquad j \mapsto \beta(j) = \left(\widehat{\beta}_{i|1}, \dots, \widehat{\beta}_{i|6}\right),$$

$$\gamma(\cdot) : \{1, \dots, 6\} \to \mathbb{R}, \qquad m \mapsto \gamma(m) = \gamma_m.$$

- Output: $\mu_{i,j,m}^{\mathrm{LoB}} = \exp\left\{\widehat{\alpha}_{i|m} + \widehat{\beta}_{j|m} + b + \langle \boldsymbol{w}, \boldsymbol{z}^{(3)}(i, j, m)\rangle\right\}$

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(iv)	multiple LoB reserves	40'271	37'027	16'400	70'563	73'314	30'730	268'305
(v)	bias CL/ccODP	-2.8%	-4.3%	-7.0%	-5.7%	-3.3%	-5.5%	-4.5%
(vi)	bias bCCNN	-1.1%	-3.1%	-6.3%	-2.0%	-2.2%	-1.4%	-2.3%
(vii)	bias multiple LoB	1.5%	0.0%	-2.8%	-1.5%	1.1%	-1.2%	-0.2%

Prediction Uncertainty (with Bootstrap)

- Conditional root mean square error of prediction (rmsep):

$$\begin{aligned}\mathrm{rmsep}\left(R^{\mathrm{true}}, R^{\mathrm{ODP}} \,\middle|\, \mathcal{D}_I\right) &= \sqrt{\mathbb{E}\left[\left(R^{\mathrm{true}} - R^{\mathrm{ODP}}\right)^2 \,\middle|\, \mathcal{D}_I\right]} \\ &= \sqrt{\mathrm{Var}\left(R^{\mathrm{true}} \middle| \mathcal{D}_I\right) + \left(R^{\mathrm{ODP}} - \mathbb{E}\left[R^{\mathrm{true}} \middle| \mathcal{D}_I\right]\right)^2}\end{aligned}$$

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- Results:

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(i)	rmsep CL/ccODP	1'076	1'316	475	2'150	1'938	975	3'528
(ii)	bias CL/ccODP	-1'120	-1'577	-1'186	-4'056	-2'382	-1'708	-12'029
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(iii)	rmsep bCCNN	1'171	1'299	508	2'105	2'029	1'072	3'607
(iv)	bias bCCNN	-456	-1'138	-1'063	-1'411	-1'612	-446	-6'126
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(v)	rmsep multiple LoB	1'102	1'357	498	2'098	1'989	1'033	3'757
(vi)	bias multiple LoB	582	-10	-478	-1'067	766	-387	-594

Relative Model Differences

- For each cell (i, j):

$$\frac{\widehat{\mu}_{i,j,\cdot}^{\mathrm{LoB}} - \widehat{\mu}_{i,j}^{\mathrm{ODP}}}{\widehat{\mu}_{i,j}^{\mathrm{ODP}}}$$

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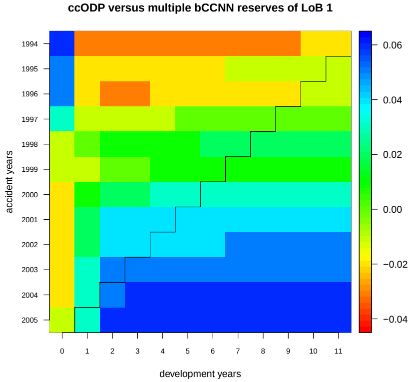

$\Longrightarrow$ slower payout pattern in more recent accident years

Cumulative Development Factors

- ccODP model:

$$f_j^{\mathrm{ODP}} = \frac{\sum_{l=0}^{j} \widehat{\mu}_{i,l}^{\mathrm{ODP}}}{\widehat{\mu}_{i,0}^{\mathrm{ODP}}} = \frac{\sum_{l=0}^{j} \exp\left\{\widehat{\beta}_l\right\}}{\exp\left\{\widehat{\beta}_0\right\}}$$

- Multiple LoB model:

$$f_{i,j}^{\mathrm{LoB}} = \frac{\sum_{l=0}^{j} \widehat{\mu}_{i,l,\cdot}^{\mathrm{LoB}}}{\widehat{\mu}_{i,0,\cdot}^{\mathrm{LoB}}}$$

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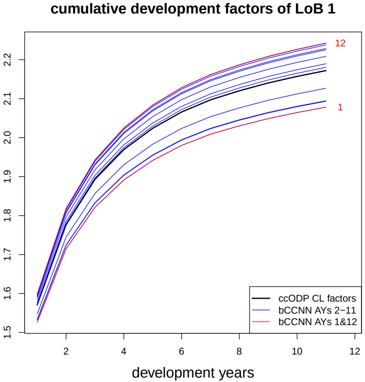

$\Longrightarrow$ accident year dependent cumulative development factors

Bias (100 Datasets)

- Simulate 100 datasets from Individual Claims History Simulation Machine
- 100 biases for:
  - ccODP/CL model
  - bCCNN model
  - multiple LoB model

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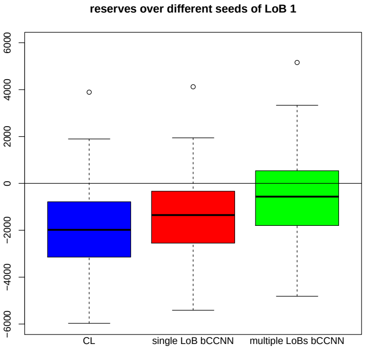

$\Longrightarrow$ learning additional model structure

Conclusions

- Learning additional model structure through embedding
- Number of training iterations has to be chosen carefully
- Small number of iterations allows us to apply bootstrap
- Extension: embedding of numbers of claims and payments

References

1. Gabrielli, A. (2019). A neural network boosted double over-dispersed Poisson claims reserving model. *SSRN Manuscript*, ID 3365517.
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4. Wüthrich, M.V., Merz, M. (2019). Editorial: Yes, we CANN! *ASTIN Bulletin* **49/1**, 1-3.